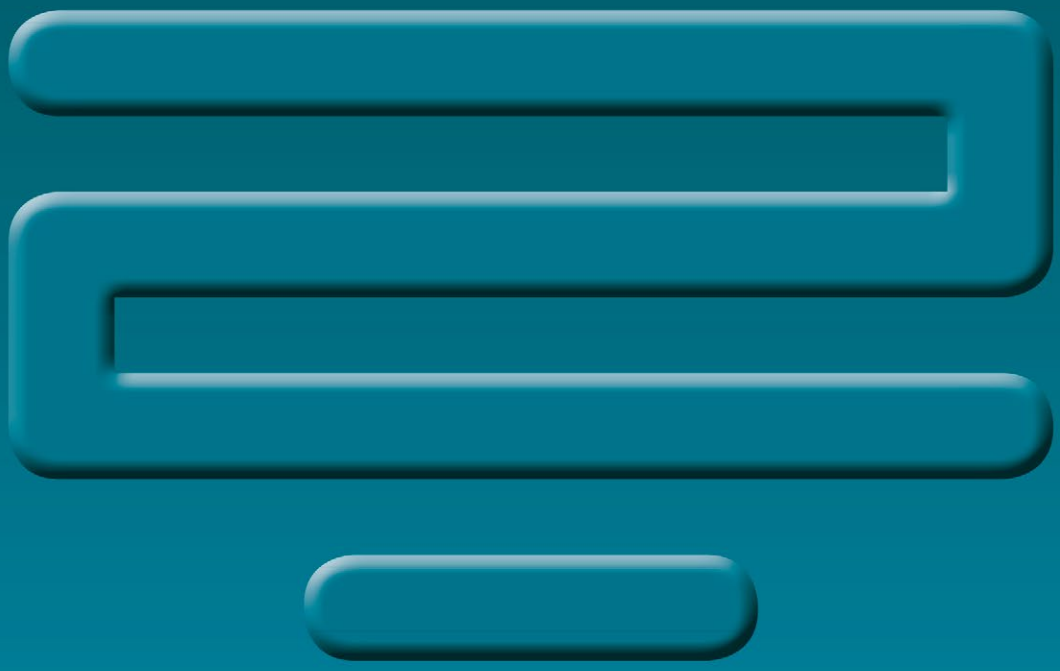
# INFERENCES WITH IGNORANCE: LOGICS OF QUESTIONS

## INFERENTIAL EROTETIC LOGIC & EROTETIC EPISTEMIC LOGIC



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**Inferences with Ignorance: Logics of Questions**
Inferential Erotetic Logic & Erotetic Epistemic Logic

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Published by Charles University in Prague
Karolinum Press
Designed by Jan Šerých
Typeset by studio Lacerta (www.sazba.cz)
First edition



ISBN 978-80-246-3181-3
ISBN 978-80-246-3193-6 (online : pdf)

Univerzita Karlova v Praze
Nakladatelství Karolinum 2015

www.karolinum.cz
ebooks@karolinum.cz

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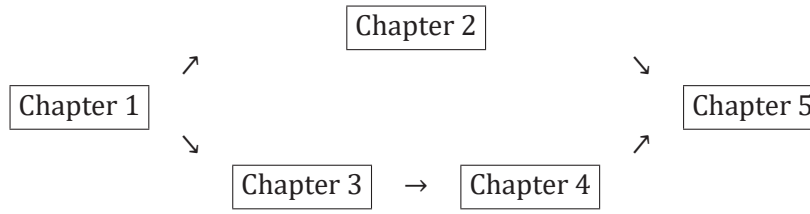
## PREFACE

As the title indicates, the book is about logic of questions (erotetic logic) which is a branch of non-classical logic. The text goes in for two formal (logical) systems of questions with the important role of erotetic inferential structures. The first one, *inferential erotetic logic*, had been originally developed in Poland in 1990s. I used this system in a slightly rearranged version that serves as the main inspiration for erotetic logic studied in the epistemic framework. The second system, *erotetic epistemic logic*, is just such a combination of epistemic logic with questions that is open for application in public announcement logic.

The text is based on my doctoral dissertation [36] which was finished during the year 2010. I decided to change some parts with respect to reviews and discussions with my colleagues and students. Nonetheless, the structure and main results are almost the same. I tried to incorporate all objections as well as the advice of my referees or, at least, I comment on them in the last chapter. The aim of this book is rather different from the thesis, therefore I checked allof the proofs and explanations and amended them. Although I believe that the text is more comprehensive now, some background knowledge of the reader is expected. A knowledge of elementary notions from formal (mathematical) logic and a basic knowledge of modal logic is assumed, and would be very helpful in reading the part starting at Chapter 3. Nice introductions to (modal) epistemic logic are introductory chapters in [48] and [11].

## STRUCTURE OF THE BOOK

The book includes two main parts that can be read independently. The first one is Chapter 2 and the second one consists of Chapters 3 and 4. Chapter 1 serves as an introduction and provides common a methodology for both parts. The last Chapter 5 contains some final remarks on the methodology used, summarizes the main results, problems, related approaches, and also further directions of the branch.




If we compare the contents of the chapters, we may find another division of the book. Both Chapter 2 and Chapter 3 can be understood as a study of the 'logic of questions'. There we are interested in basic erotetic structures,

i.e., inferences with questions, relationships of questions and declaratives, and answerhood conditions. However, Chapter 4 introduces questions as a part of communication; a dynamic approach is applied in the erotetic epistemic framework introduced.[1]

## CHAPTER 1: LOGIC AND QUESTIONS

The chapter introduces a multi-paradigmatic situation in the methodology of erotetic logic and contains a short historical overview of this branch with a special emphasis on recent developments.[2] We briefly introduce *inferential erotetic logic*, Groenendijk-Stokhof's intensional approach, and some developments of these theories. However, the core of the chapter is devoted to a formalization of questions based on sets of answers. We justify the usefulness of the set-of-answers methodology in the study of erotetic consequence relations as well as in an epistemic interpretation of questions.

This chapter is based on the paper [37].

## CHAPTER 2: CONSEQUENCE RELATIONS IN INFERENTIAL EROTETIC LOGIC

This part is aimed at the studying of relationships among consequence relations that are introduced in *inferential erotetic logic* (IEL). We keep the framework of IEL, but the question representation uses the methodology from Chapter 1. IEL requires that declarative and interrogative formulas are not mixed on the object-language level; answers are strictly declarative sentences. The defined consequence relations with questions are naturally based on the multiple-conclusion entailment among sets of declarative formulas. We add the term *semantic range* of a question to the terminology of IEL and work with sets of declaratives as associated with classes of models. This 'model-based approach' makes proofs and properties very transparent. The chapter is a technical overview of some IEL concepts and their properties. We understand the IEL presented as providing a general framework and inspiration for the work with inferences among questions and declaratives.

Chapter 2 can be read as a full introduction to the topic; no reading of another text is required. The chapter was originally published in [35].

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1 Dynamic-like approaches bear the name 'logic of inquiry' in literature.

2 Approximately till the year 2010. Comments on the newest literature are in Chapter 5.

## CHAPTER 3: EPISTEMIC LOGIC WITH QUESTIONS

The main goal here is to incorporate questions in a general epistemic framework. Questions are represented by finite sets of direct answers and their 'satisfiability' in a state of an epistemic model is based on three conditions that express ignorance and the presuppositions of a questioner.[3] In the framework of normal modal logic, a question becomes a complex modal formula. Inspired by inferential structures in IEL, we show that there are 'philosophically' similar structures based on a classical implication. The rest of the chapter is devoted to answerhood conditions and the role of the implication with respect to the epistemic context and conjunctions of yes-no questions.

This chapter cannot be read without a basic knowledge of modal (epistemic) logic.

## CHAPTER 4: A STEP TOWARDS THE DYNAMIZATION OF EROTETIC LOGIC

This chapter takes full advantage of the multi-agent extension of the setting from Chapter 3 and can be considered an application of the introduced erotetic-epistemic approach in a dynamic framework. We define here *public announcement logic* based on the S5 modal system extended by group modalities. The askablity of questions as well as answerhood conditions are studied from the viewpoint of groups of agents. As an application, we show the role of questions and group modalities in a (communication-like) answer 'mining' in a group of agents.

Some results of the chapter were published in [38, 39].

## CHAPTER 5: CONCLUSION

The last chapter includes a brief overview of related works, main results, possibilities of future directions, and comments on the problematic parts of our approach. This chapter is the result of many discussions. The important part of the subsection devoted to weak points and problems is based on the reviews to the original doctoral thesis. A list of up-to-date publications cannot be complete. However, we tried to add important papers closely related to our approach.

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3 We will use the term *askability* of a question (for an agent in a state).

## ACKNOWLEDGEMENTS

It would be a long list of names to express my thanks to everybody who assisted me in the work on erotetic logic. And it was many names and grants I listed in the preface to my doctoral dissertation [36]. Therefore, now, I will mention especially people, they helped me in the preparation of the book.

The great help, with an improvement of the text, came from my thesis referees Andrzej Wiśniewski and Igor Sedlár. They carefully reviewed my dissertation and gave me a lot of helpful comments and recommendations. Many of them were worked in. Andrzej as well as Igor supported me with their papers and advice. Igor together with Mariusz Urbański took on board the review of this book and helped me to prepare the final version. Thank you.

My special thanks go to Ondrej Majer who has helped me with the work in dynamization of erotetic epistemic logic. We wrote some papers from this field and, simultaneously, we have been working in epistemic relevant logic. We have been organizing seminars on dynamic logics for some years. The audience of the seminar was the best test of my ideas; let me mention particularly Marta Bílková, Michal Dančák, Adam Přenosil, Vít Punčochář, and Petr Švarný.

Last but not least, my thanks go to my colleagues and friends at the Faculty of Arts and at the Institute of Philosophy. Let me mention, especially, Lenka Jankovská, Paweł Łupkowski, Jaroslav Peregrin, Vladimír Svoboda, and Vítězslav Švejdar.

The work on this book was supported by the Institute of Philosophy of the Czech Academy of Sciences and by the Program for the Development of Fields of Study at Charles University, No. P13 Rationality in Human Sciences, sub-program 'Modern logic, their methods and applications'.

# 1. LOGIC AND QUESTIONS

## 1.1 QUESTIONS, ANSWERS, AND INFERENCES

In this chapter we wish to show that it is reasonable to consider questions as a part of logical study. In logic, declarative sentences usually have their formal (logical) counterparts and play an important role in argumentation. We often see logic to be primarily a study of inferences. Inferential structures are studied in formal systems, which can differ in the formalization of declaratives as well as in admitting or rejecting some principles.

We believe that dealing with questions in the logical framework will be justified if we show that questions can play an autonomous and important role in inferences. Perhaps this point may be considered as the most important to justify *logic of questions*.[1]

This introductory chapter provides a brief overview of the history as well as methodology in recent approaches to erotetic logic. However, the main aim is to concentrate on the methodology used in the rest of the book—we introduce and discuss a variant of the methodology based on sets of answers.

1 In this paper we use the term *logic of questions* in the same meaning as *erotetic logic*, discussions on both terms can be found in [19].

### 1.1.1 QUESTIONS AND ANSWERS

Let us imagine a group of three friends: Anne, Bill, and Catherine. Each of them has one card and nobody can see the cards of the others. One of the cards is the Joker and everybody knows this fact.[2] Then

Who has the Joker?

is a reasonable sentence in this situation. We recognize it as an *interrogative sentence* because of its word order and the question mark. Moreover, interrogatives are often connected with intonation and interrogative pronunciation.

An interrogative sentence includes more—a pragmatic aspect. A question is a "request to an addressee to provide the speaker with certain information", this is an *interrogative speech act* [17, p. 1057]. Pragmatically oriented approaches emphasize the roles of a speaker and an addressee. The roles seem, on the one hand, to be outside of the proper meaning of interrogatives. On the other hand, they are often understood to be a crucial aspect in analyses of questions; this can be the reason why some logicians argue against some variants of the logic of questions.

No matter what our starting position is if we want to work with interrogatives in the framework of a formal system, we are obliged to decide the following two problems at least:

1. How to formalize questions?
2. What is the (formal) semantics of questions?

Reviewing the history of erotetic logic, there is no unique solution. There are many approaches to the formalization of questions and every approach varies according to what is considered important. Logic of questions is multiparadigmatic. David Harrah [18, pp. 25–26] illustrates it nicely with examples of so called 'meta-axioms'. He groups them into three sets according to the degree of acceptance by erotetic logicians:

1. The first group includes meta-axioms accepted in almost all systems. Harrah calls them *absolute axioms*. For example:
   (a) Every question has at least one partial answer.
   (b) (In systems with negation) For every statement $P$, there exists a question $Q$ whose direct answers include $P$ and the negation of $P$.
   (c) Every question $Q$ has a presupposition $P$ such that: $P$ is a statement, and if $Q$ has any true direct answer, then $P$ is true.

---

2 In the epistemic setting we will expect more: the rules of a 'game' are *commonly known*, cf. Chapter 4.

2. The second group, *standard axioms*, is often accepted, but not in all systems:
    (a) Every question has at least one direct answer.
    (b) Every direct answer is a statement.
    (c) Every partial answer is implied by some direct answer.
    (d) Every question is expressed by at least one interrogative.
    (e) Each interrogative expresses exactly one question.
    (f) Given an interrogative $I$ there is an effective method for determining the direct answers to the question expressed by $I$.
3. The last group is called *eccentric axioms*. The following examples of such axioms are accepted only in some interrogative systems:
    (a) If two questions have the same direct answers, then the two questions are identical.
    (b) Every question $Q$ has a presupposition that is true just in case some direct answer to $Q$ is true.

Let us notice the terminology, the difference between *interrogative (sentence)* and *question* has been just introduced by standard axioms. The first term mostly refers to the type of a sentence and the second one is a bit more complex. A question is expressed by an interrogative (sentence) and can be 'posed', 'asked', etc., cf. [19]. Although we use *interrogative* and *question* in the same meaning here, the term *interrogative sentence* is reserved for a natural-language sentence, if necessary.

What seems to be common to all approaches is that questions are something structured and closely connected with their answers. We hardly find a propositional theory where questions are unstructured as atomic propositions are. The meaning of a question is always closely connected to its *answerhood conditions*.

Since an answer to a question is often represented by a declarative, the natural starting point of many erotetic theories is a standard formal system for declaratives.

> "Any first-order language can be supplemented with a question-and-answer system" [51, p. 37].

This broadly accepted statement invites us to solve the problem with the formalization of questions together with their meaning. Questions' autonomy depends on the chosen solution. Andrzej Wiśniewski distinguishes two basic groups of erotetic theories: *reductionist* and *non-reductionist*. Roughly speaking, non-reductionism is characterized by the claim that questions "are not reducible to expressions of other syntactic categories" [51, p. 40], see Section 1.2.1 in this chapter, too. The boundary between both groups is fuzzy. Perhaps only pure

pragmatically oriented approaches belong to the radical reductionism with a complete rejection of questions as a specific entity in formal logic.[3]

### 1.1.2 INFERENCES WITH QUESTIONS

Although there are discussions on whether it is necessary to work with questions as a new specific entity in a formal system, almost all theorists agree that questions play a specific role in inferences. Let us come back to our group of friends. The situation, where

> *either Anne has the Joker or Bill has the Joker or Catherine has the Joker*,

can raise the question

> $Q$: *Who has the Joker?*

A question is raised (inferred) from a declarative or from a set of declaratives. What would make this raising reasonable? 'Answerhood conditions' is the answer. Any 'reasonable' answer to the question $Q$ is connected to the declarative context.

Another kind of inferential structure could be based on declaratives as well as questions among premises. For example, from

> $Q$: *Who has the Joker?*

and

> $\Gamma$: *The only person from London has the Joker.*

can be inferred the question

> $Q_1$: *Who is from London?*

The relationship of the inferred question $Q_1$ and the initial question $Q$ is based on their answerhood conditions. An answer to $Q$ can provide an answer to $Q_1$ with respect to the context $\Gamma$. Moreover, in this example, $Q$ can be inferred from $Q_1$ with $\Gamma$ as well. This shows that the relationship is dependent on various kinds of answerhood conditions and contexts.

Let us have $Q$ remain the same, but the context is

> *A person from London has the Joker.*

---

3 An example of one of the radical approaches [43] is commented in [34].

If two persons are from London and we gain their names in an answer to $Q_1$, then we receive only a *partial answer* to $Q$.[4] If each of the friends (or nobody) is from London, an answer to $Q_1$ does not provide any help for the answering of $Q$. Then we can discuss the utility of an inferential relation between $Q$ and $Q_1$ with respect to this context.

The role of answerhood conditions in inferences among questions is obvious in the following example: From any (complete) answer to

*Who has the Joker?*

we obtain a (complete) answer to the question

*Does Anne have the Joker?*

as well as to the questions

*Does Bill have the Joker?*

and

*Does Catherine have the Joker?*

Answerhood conditions of the previous three questions are *entailed* in the answerhood conditions of the question $Q$. They can be inferred from the answerhood conditions of $Q$. The question *'Does Anne have the Joker?'* is *entailed* by *'Who has the Joker?'*.

We have just presented inference-like structures with questions as dependent on answerhood conditions. Now, still faced with the problem at how to formalize the relationship of questions and answers, we will introduce a convenient solution based on a liberal set-of-answers methodology.

## 1.2 SET-OF-ANSWERS METHODOLOGY

We are going to solve the problem of the formal shape of questions simultaneously with the problem of the questions' semantics. The formalization of a question will be based on a (possibly infinite) set of 'specific' answers. Moreover, we attempt to show that such an approach can also satisfy some semantic and pragmatic requirements.

---

4 Informally, a *partial answer* does not completely answer a question, but it eliminates some of the possible (and complete) answers. The term *possible and complete answer* expresses that the answer 'completely answers' a question. Later on we will introduce formal definitions.

### 1.2.1 SEMANTICS OF QUESTIONS

Some theories do not admit that questions could have an independent meaning in logic. Questions are paraphrased by declarative sentences; in particular, the question *'Who has the Joker?'* may be then paraphrased by

*I ask you who has the Joker.*

Another way is the paraphrasing by epistemic-imperative sentences:

*Bring it about that I know who has the Joker!*

The adequacy of both paraphrases in capturing the complete meaning of a question is rather problematic.[5] One of the problems is that these approaches are forced to work with a questioner and an addressee already on the basic level of the questions' meaning. Of course, we expect to utilize the importance of a questioner and an addressee, but it should be a task for a chosen background system not for the general semantics of questions.[6] We understand pragmatic aspects as a higher level analysis.

Nuel Belnap formulated three methodological constraints on the meaning of questions which he used for classification and evaluation of erotetic theories:[7]

1. **Independence** Interrogatives are entitled to a meaning of their own.
2. **Equivalence** Interrogatives and their embedded forms are to be treated on a par.
3. **Answerhood** The meaning of an interrogative resides in its answerhood conditions.

The most important is the first requirement, which is the main sign of non-reductionist theories. To accept the *independence* requirement means that we are obliged to look for the specific semantics of questions. The *equivalence* requirement is closely related to a semantic entailment and is dependent on the chosen semantics. *Answerhood* requires that the meaning of questions is related to the meaning of answers. In addition, we can work with the idea that the semantics of answers forms a good background for the study of the meaning of interrogatives.

Approaches, which accepts that answers are crucial for the meaning of questions, are in compliance with the first postulate from the following list suggested by Charles Hamblin:[8]

---

5 See also [19] for other examples and references.

6 In chapters 3 and 4 we will follow this idea and the background system will be dynamic epistemic logic.

7 Belnap, N.D., 'Approaches to the semantics of questions in natural language. Part I', Pittsburgh, 1981. Cited from [16, p. 3–4].

8 Hamblin, C.L., 'Questions'. Australasian Journal of Philosophy, 36(3): 159–168, 1958. Cited from [19].

1. Knowing what counts as an answer is equivalent to knowing the question.
2. An answer to a question is a statement.
3. The possible answers to a question are an exhaustive set of mutually exclusive possibilities.

Each postulate may be argued against and a detailed discussion is available in [17]. However, according to David Harrah, adopting the first one is "the giant step toward formalization often called *set-of-answers methodology*" [19, section 2]. Although there is not only one kind of set-of-answers methodology (SAM, for short) in the literature, we will not make any survey here. In the next subsection we introduce an easy idea of a question representation by a set of *direct answers*.

### 1.2.2 SETS OF ANSWERS

Generally, without any context, the question *Who has the Joker?* can be answered by expressions of the following form:

*Anne.*
*Anne has it.*
*Anne has the Joker.*
*Anne and Bill.*
⋮
*Batman has the Joker.*
⋮
*Your friends.*
*People at this table.*
⋮
*Nobody.*
⋮
etc.

The question seems to be answered if a (complete) list of Joker owners is given. We can assume that answers are propositions; thus, the first three items in the list have the same meaning in the answering of the question.

From the viewpoint of propositional logic and in accordance with the first two of Hamblin's postulates, we can understand every question as closely connected with a set of (propositional) formulas—formalized answers.

Furthermore, we can receive some of the following responses to the same question:

*Anne doesn't have the Joker.*

or

*I don't know who has the Joker.*

The first one can be considered to be a *partial answer*; it removes some answers as false, in particular, all answers with Anne having the Joker.[9]

The second one appears to bear another kind of information; an addressee says to a questioner that she has the same problem and would ask the same question. (We will return to this topic in the last paragraph of Section 1.4.)

If we had decided to represent every question by a complete set of its answers, we would not always have a clear and useful formalization of questions. Let us return to our example of three friends with cards. Considering the context and the question *Who has the Joker?*, a questioner expects one of the following responses:

$\alpha$: *Anne has the Joker.*
$\beta$: *Bill has the Joker.*
$\gamma$: *Catherine has the Joker.*

or a response that leads to one of the just mentioned. In fact, the question

*Who has the Joker?*

with respect to the context

*Either Anne has the Joker or Bill has the Joker or Catherine has the Joker.*

might be reformulated to

$Q'$: *Who has the Joker: Anne, Bill, or Catherine?*

The answers $\alpha$, $\beta$, and $\gamma$ are understood as 'core' answers that form the meaning of the question $Q'$. We use the term *direct answers* for them. The sentence

$\delta$: *Neither Anne nor Bill have the Joker.*

is an answer, from which the answer $\gamma$ can be inferred thanks to the context. We call $\delta$ *complete answer* to $Q'$. Complete answers are 'solutions' of a question and the set of direct answers is a subset of the set of complete ones.

Our SAM is inspired by the syntactic representation of questions in *inferential erotetic logic* founded by Andrzej Wiśniewski.[10] We want to be very liberal and, thus, to represent questions as sets of formulas, which play the role

9 We can imagine a context when the answer is complete—only two players.

10 The best overview of questions' formalization in inferential erotetic logic is in the book [51, chapter 3]. See also the article [53] and the new book [59]. We will return to this system in Chapter 2.